

Splitting of the superconducting transition in the two weakly coupled 2D XY models

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The frequency ω and temperature T dependent complex conductivity σ of two weakly coupled 2D XY models subject to the RSJ dynamics is studied through computer simulations. A double dissipation-peak structure in $\text{Re}[\omega\sigma]$ is found as a function of T for a fixed frequency. The characteristics of this double-peak structure, as well as its frequency dependence, is investigated with respect to the difference in the critical temperatures of the two XY models, originating from their different coupling strengths. The similarity with the experimental data in Festin *et al.* [Physica C **369**, 295 (2002)] for a thin YBCO film is pointed out and some possible implications are suggested.

The thermally excited two-dimensional (2D) vortex fluctuations drive the phase transition between the superconducting and normal state in many 2D systems. Such transition is of the Kosterlitz-Thouless (KT) type: below the KT transition temperature T_c vortices are bound in pairs with total vorticity zero, and as the temperature is increased across T_c from below these vortex pairs start to unbind [1]. It means that the dominant characteristic physical features in a region close to the KT transition are associated with vortex pair fluctuations. One of the physical quantities which contain the information about the feature of vortex dynamics is the frequency ω dependent complex conductivity $\sigma(\omega)$ of the sample [2,3]. In the presence of 2D fluctuation effects $\sigma(\omega)$ can be expressed as $\sigma(\omega) = -\rho_0(T)/i\omega\epsilon(\omega)$, where $1/\epsilon(\omega)$ is the dynamic dielectric function which describes the effect of pair motion, and $\rho_0(T)$ is the bare superfluid density [2,4]. The measurements of the superconducting transition by means of $\sigma(\omega)$ in a typical experiment for a fixed frequency should show a single peak in $\text{Re}[\omega\sigma(\omega)]$ at a frequency dependent temperature T_ω . This peak represents dissipation losses while a sharp decrease in $\text{Im}[-\omega\sigma(\omega)]$ at the same temperature is a consequence of the loss of a superfluid response. Indeed, such behaviors of $\sigma(\omega)$ have been confirmed in many experiments on 2D superconductors, as well as on high- T_c superconductors [3,5].

The present investigation is inspired by the recent experimental results obtained by Festin *et al.* [6] for a 1500 Å thin YBCO film: A very striking double peak structure in $\text{Re}[\omega\sigma(\omega)]$ is found; two rapid drops of $\text{Im}[-\omega\sigma(\omega)]$ at different T are observed. The data by Festin *et al.* are reproduced in Figs. 1 and 2. One possible explanation for the double peak is that the epitaxial grown YBCO film is split into an upper and lower part due to a slightly different oxygen contents [7]. From this perspective the sample would consist of two weakly coupled parallel superconducting parts.

In order to investigate this scenario we use the two weakly coupled 2D XY models, the Hamiltonian of which is written as

$$\begin{aligned} H &= H_1 + H_2 + H_{\text{int}}, \\ H_1 &\equiv -J_1 \sum_{\langle ij \rangle} \cos(\theta_i^{(1)} - \theta_j^{(1)}), \\ H_2 &\equiv -J_2 \sum_{\langle ij \rangle} \cos(\theta_i^{(2)} - \theta_j^{(2)}), \\ H_{\text{int}} &\equiv -J_\perp \sum_i \cos(\theta_i^{(2)} - \theta_i^{(1)}), \end{aligned}$$

where H_1 and H_2 are the usual 2D XY Hamiltonians with the coupling strengths J_1 and J_2 for the first (lower) and the second (upper) planes, respectively, the summation $\sum_{\langle ij \rangle}$ is over all nearest neighbor pairs in each plane, and H_{int} with the coupling strength J_\perp describes the coupling

between the planes. To study the dynamics of the system, we use the equations of motion of the standard resistively-shunted junction (RSJ) dynamics subject to the periodic boundary condition and integrate the equations of motion using the second-order algorithm with the time step $\Delta t = 0.05$. We also apply the fast Fourier transformation method to speed up the calculations (see e.g., Ref. [8] for details).

The 2D XY model on the square lattice undergoes a KT transition at $T_c \approx 0.89J$ [9], where J is the Josephson coupling strength. Accordingly, two planes with different coupling constants then undergo two separate phase transitions at the different temperature $T_c \approx 0.89J_1$ and $0.89J_2$ when the interplane coupling vanishes. The main output from the simulation are the dynamic dielectric function $1/\epsilon(\omega)$, and the helicity modulus γ , which basically measures the stiffness of the system to a twist in the phase of the order parameter and is proportional to the renormalized superfluid density $\rho = \rho_0/\epsilon(0)$ [2]. From a knowledge of $1/\epsilon(\omega)$ and ρ_0 it is straightforward to analyze the behavior of the conductivity $\sigma(\omega)$.

In order to get realistic parameters for the two weakly coupled 2D XY models in connection with the experimental result by Festin *et al.* one would like to have some reasonable estimates of the coupling constants J_1 , J_2 , and J_\perp . We estimate J_1 and J_2 from the experimental data reproduced in Fig. 1. In order to do it we use the relation $\lim_{\omega \rightarrow 0} \text{Im}[-\omega\sigma(\omega)] = \rho_0/\epsilon(0)$ and note that $1/\epsilon \approx 1$ just below the transition and is 0 just above. Thus $\rho_0 \propto J$ [1] may be roughly estimated by the two heights of the rapid drops in Fig. 1 for the curve corresponding to the smallest frequency. From this we get the ratio $J_2/J_1 \approx 0.5$ (ratio between the dashed vertical lines in Fig. 1). The corresponding curves for $\text{Re}[\omega\sigma(\omega)]$ are shown in Fig. 2, where the characteristic double peaks at different temperatures are clearly exhibited. J_\perp can be estimated from the knowledge of the anisotropy parameter Γ defined as $\Gamma \equiv \sqrt{J_1/J_\perp}$ and found to be equal to 7 for YBCO [10,11].

First we present the result for our model without coupling between the two planes. $J_\perp = 0$ corresponds to the case of no supercurrent flowing between the planes. Figures 3 and 4 show the

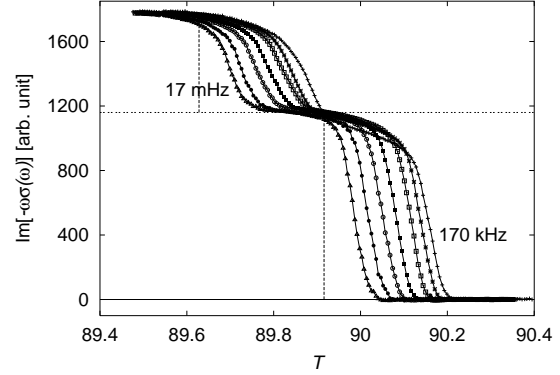


Figure 1. Experimentally measured $\text{Im}[-\omega\sigma(\omega)]$ vs T for $\omega = 17$ mHz, 170 mHz, 1.7 Hz, 17 Hz, 170 Hz, 1.7 kHz, 170 kHz (from the left to the right). Horizontal lines shows respective zero levels for transitions and vertical lines indicates the heights of the rapid drop of $\text{Im}[-\omega\sigma(\omega)]$ which gives a rough measure of the superconducting electrons involved. The data are taken from Ref. [6].

behavior of $\text{Re}[\omega\sigma(\omega)]$ and $\text{Im}[-\omega\sigma(\omega)]$ obtained from the simulations with $J_1 = 1$, $J_2 = 0.5$ and $J_\perp = 0$. The similarities to the experimental results in Figs. 1 and 2 are striking: $\text{Re}[\omega\sigma(\omega)]$ in Fig. 4 again displays two distinct peaks at different temperatures while $\text{Im}[-\omega\sigma(\omega)]$ in Fig. 3 has two regions with increased drops as a function of T . Dashed line in Fig. 3 represents the behavior of γ or equally the behavior of $\text{Im}[-\omega\sigma(\omega)]$ in the limit $\omega \rightarrow 0$ which shows one drop at $T_{c1} \approx 0.89$ and another at $T_{c2} \approx 0.45$. These similarities between experiments and our simulations for the coupled 2D XY models are further substantiated when one compares the frequency dependences: As the frequency is increased both T_{c1} and T_{c2} increase as is reflected in the positions of peaks in Figs. 2 and 4. The soundness of the method of estimation of the coupling constants J_1 and J_2 is illustrated in Fig. 3 where the helicity modulus is represented by the dashed line. Following the suggestion that J_2/J_1 can be estimated from the ratio between the dotted vertical lines we again get $J_2/J_1 \approx 0.5$. The quantitative differences be-

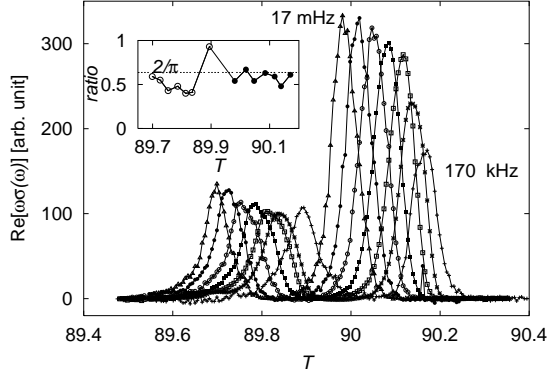


Figure 2. $\text{Re}[\omega\sigma(\omega)]$ vs T measured for a thin YBCO film at the same frequencies as $\text{Im}[-\omega\sigma(\omega)]$ in Fig.1. The inset illustrates the peak ratio defined as the ratio $\text{Re}[\omega\sigma(\omega)]/\text{Im}[-\omega\sigma(\omega)]$ taken at the dissipation peak maxima for a given ω . The data are taken from Ref. [6].

tween the data in Fig. 2 and the simulations in Fig. 4 is according to our interpretation due to the fact that the simulations cannot reach as low frequencies as the experiments (more precisely the ratio between the frequency and the microscopic frequency scale is larger in the simulations). A higher frequency broadens and smears the dissipation peaks. Since the interplane coupling in experiments presumably does not vanish, we have also investigated the influence of a small J_{\perp} in our model and found that a weak interplane coupling ($0 \leq J_{\perp} \leq 0.04$) does not influence the qualitative features found above for $J_{\perp} = 0$.

The similarity with the experimental data by Festin *et al.* implies that the YBCO film is split parallel to the surface into two superconducting parts both having transitions with 2D character but with different T_c . Since the coupling between the planes does not wash away the double peak transition one must infer that the coupling between the two sheets is very weak. One possible reason for the phase separation may lie in a combination of an inhomogeneous surface with a more homogeneous part closer to the substrate. This can lead to different oxygen contents in the two

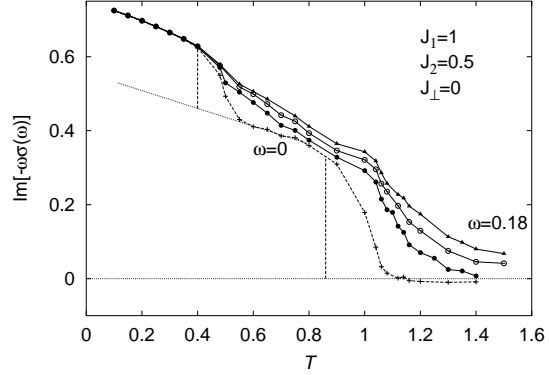


Figure 3. $\text{Im}[-\omega\sigma(\omega)]$ vs T for two weakly coupled 2D XY models with system size $L = 64$ at $\omega = 0, 0.06, 0.12, 0.18$ (from the left to the right). The coupling constants are chosen to be $J_1 = 1$, $J_2 = 0.5$, and $J_{\perp} = 0$.

parts, which in turn results in slightly different lattice parameters causing a physical boundary between the two parts [6,12–14]

A further consistency check on the scenario in terms of two coupled 2D superconducting parts comes from the peak ratio defined as the ratio $\text{Re}[\omega\sigma(\omega)]/\text{Im}[-\omega\sigma(\omega)]$ taken at the dissipation peak maxima for a given ω . For a 2D superconductor this peak ratio should vary between $2/\pi \approx 0.63$ for small ω to 1 for larger ω [15]. The inset in Fig. 2 gives the ratios estimated from the experimental data. The filled circles is for the large- T peaks and these ratios are consistent with 2D vortex fluctuations close to the transition. The empty circles corresponds to the small- T peaks where in accordance with an interpretation in terms of two transitions the zero level of $\text{Im}[-\omega\sigma(\omega)]$ for the low- T transition is estimated by a linear extrapolation of the large T -part towards lower T . An almost as good estimate is to approximate by the plateau between the transitions (horizontal line in Fig. 1). These peak ratios are also rather consistent with a 2D transition in the limit of small ω (peak ratios for the lowest- T values). The small deviation for higher temperatures may be due to a convolution of the high and low T parts of the transition.

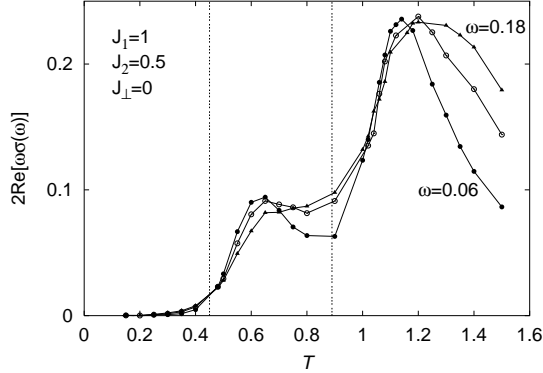


Figure 4. $\text{Re}[\omega\sigma(\omega)]$ as a function of temperature for different ω : the lines with filled circles, empty circles and triangles correspond to the frequency 0.06, 0.12, and 0.18, respectively. Vertical lines show T_c for two decoupled 2D XY models with coupling constants $J_1 = 1$ and $J_2 = 0.5$: $T_{c1} \approx 0.89$ and $T_{c2} \approx 0.45$.

Another question is at what thickness d the division takes place. A rough estimate may be obtained by assuming that both sheets consist of identical material. In such a case $J_1 \propto d_1 n_S$ and $J_2 \propto d_2 n_S$ where $d_{1(2)}$ is the thickness of respective sheets and n_S is the density of Cooper pairs for the material. Thus $J_2/J_1 \approx d_2/d_1$ and since we have found that $J_2/J_1 \approx 0.5$ from the data, we conclude that the boundary between the two parts should occur somewhere in the middle of the sample.

In summary we conclude from our numerical simulations of two weakly coupled XY models that the double peak dissipation structure observed by Festin *et al.* is consistent with the interpretation that the sample consists of two parts with slightly different transition temperatures, and that these two parts are separated by a boundary which mainly runs parallel to the substrate.

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